Ordering dynamics with two non-excluding options: Bilingualism in language competition

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Abstract: We consider a modification of the voter model in which a set of interacting elements (agents) can be in either of two equivalent states (A or B) or in a third additional mixed AB state. The model is motivated by studies of language competition dynamics, where the AB state is associated with bilingualism. We study the ordering process and associated interface and coarsening dynamics in regular lattices and small world networks. Agents in the AB state define the interfaces, changing the interfacial noise driven coarsening of the voter model to curvature driven coarsening. We argue that this change in the coarsening mechanism is generic for perturbations of the voter model dynamics. When interaction is through a small world network the AB agents restore coarsening, eliminating the metastable states of the voter model. The time to reach the absorbing state scales with system size as $\tau \sim \ln N$ to be compared with the result $\tau \sim N$ for the voter model in a small world network.

1.INTRODUCTION

Understanding the complex collective behavior of many particle systems in terms of a microscopic description based on the interaction rules among the particles is the well established purpose of Statistical Physics. This micro-macro paradigm [1] is also shared by Social Science studies based on agent interactions (Agent Based Models). In many cases parallel research in both disciplines goes far beyond superficial analogies. For example, Schelling's model [1] of residential segregation is mathematically equivalent to the zero-temperature spin-exchange Kinetic Ising model with vacancies. Cross-fertilization between these research fields opens interesting new topics of research [2]. In this context the consensus problem is a general one of broad

interest: the question is to establish when the dynamics of a set of interacting agents that can choose among several options leads to a consensus in one of these options, or alternatively, when a state with several coexisting social options prevails [3]. For an equilibrium system the analogy would be with an order-disorder transition. For nonequilibrium dynamics we rely on ideas of studies of domain growth and coarsening in the kinetics of phase transitions [4], where dynamics is dominated by interface motion. Microscopic interaction rules include two ingredients that determine the ultimate fate of the system, either homogenous consensus state or spatial coexistence of domains of different options. These ingredients are: i) the interaction mechanism between particles/agents, and ii) the network of interactions. Interactions in complex networks is a relatively recent paradigm in statistical physics [5]. A general still open question is the study of coarsening in complex networks [6].

Language competition is a particular example of consensus problems that motivates the present work. It refers to the dynamics of language use in a multilingual social system due to individuals interacting in a social network. Recent interest in this problem has been triggered by the model proposed by Abrams and Strogatz (AS-model) [7] to account for data of extinction of endangered languages [8]. Other different problems of language dynamics include those of language evolution (dynamics of language structure) and language cognition (learning processes). Among these, *semiotic dynamics*, considered in the context of the *naming game* [9], is also an example of consensus problems. The seminal paper of Abrams and Strogatz [7], as well as others along the same line [10, 11, 12], belong to the general class of mean-field population dynamics studies based on nonlinear ordinary differential equations for the populations of speakers of different languages. Other studies implement microscopic agent-based-models with speakers of many or few languages [13, 14, 15, 16] as reviewed in [17].

The microscopic version [16] of the AS-model for the competition of two equivalent languages is equivalent to the voter model [18, 19, 20, 21, 22, 23]. The voter model is a prototype lattice spin-model of nonequilibrium dynamics for which $d = 2$ is a critical dimension [20]: For regular lattices with $d > 2$ coarsening does not occur and, in the thermodynamic limit, the system does not reach one of the homogenous absorbing states (consensus states). The same phenomenon occurs in complex networks of interaction of effective large dimensionality where a finite system gets trapped in long-lived heterogeneous mestastable states [21, 22, 23]. From the point of view of

interaction mechanisms, the voter model is one of random imitation of a state of a neighbor. A different mechanism (for $d > 1$) of majority rule is the one implemented in a zero-temperature spin-flip kinetic Ising (SFKI) model [24]. Detailed comparative studies of the consequences of these two mechanisms in different interaction networks have been recently reported [25]. From the point of view of coarsening and interface dynamics, a main difference is that, in the voter model coarsening is driven by interfacial noise, while for a SFKI coarsening is curvature driven with surface tension reduction.

The voter and SFKI models are two-option models (spin $+1$ and spin -1) with two equivalent global attractors for the system. Kinetics of multi-option models like Potts or clock models were addressed long ago [26]. More recently, a related model proposed by Axelrod [27] has been studied in some detail [28, 29]. This is a multi-option model but, in general, its nonequilibrium dynamics does not minimize a potential leading to a thermodynamic equilibrium state like in traditional statistical physics [30]. On the other hand, the kinetics of the simplest three-options models [31, 32, 33] has not been studied in great detail.

We are here interested in the class of 3-state models for which two states are equivalent (spin ± 1, state A or B) and a third one is not (spin 0, state AB). Different dynamical microscopic rules can be implemented for such choice of individual states, some of which can be regarded as constrained voter-model dynamics [32]. The choice of dynamical rules in this paper is dictated by our motivation of considering bilingual individuals in the competition dynamics of two languages [12, 13]. We will consider here two socially equivalent languages. The possible state of the agents are speaking either of these languages (A or B) or a third non-equivalent bilingual state (AB). In the context of the consensus problem this introduces a special ingredient in the sense that the options are not excluding: there is a possible state of the agents (bilinguals) in which there is coexistence of two possible options. In a more general framework, the problem addressed here is that of competition or emergence of social norms [34] in the case where two norms can coexist at the individual level.

In this paper, and building upon a proposal by Minett and Wang [13] we study a microscopic model of language competition which reduces to the microscopic AS-model [16] when bilingual agents are not taken into account. Our presentation in the remaining sections of the paper is of general nature for the abstract problem of ordering dynamics of a modified voter model in which a third mixed AB state is allowed. We aim to explore possible mech-

anisms for the stabilization of two options coexistence, possible metastable sates, and the role of AB states (bilingual individuals) and interaction network (social structure) in these processes. To this end we analyze the growth mechanisms of A or B spatial domains (monolingual domains), the dynamics at the interfaces (linguistic borders), and the role of AB states (bilingual individuals) in processes of domain growth. This is done in regular lattices and in complex networks of interaction. Generally speaking, we find that allowing for the AB state (bilinguals) modifies the nature and dynamics of interfaces: agents in the AB state define thin interfaces and coarsening processes change from voter-like dynamics to curvature driven dynamics. We argue that this change of coarsening mechanism is generic for perturbations of the voter model.

The outline of the paper is as follows: Section 2 describes our microscopic model which is analyzed in a 2-dimensional regular lattice in Section 3. In Section 4 we describe the dynamics of the model in a small world network [35]. Section 5 contains a summary of our results.

2. A MODEL WITH TWO NON-EXCLUDING OPTIONS

We consider a model in which an agent i sits in a node within a network of N individuals and has k_i neighbours. It can be in three possible states: A, agent choosing option A (using language A); B, agent choosing option B (using language B); and AB, agent in a state of coexisting options (bilingual agent using both languages, A and B). States A and B are equivalent states.

The state of an agent evolves according to the following rules: starting from a given initial condition, at each iteration we choose one agent i at random and we compute the local densities for each of the three communities in the neighbourhood of node i, σ_i (i=A, B, AB). The agent changes state according to the following transition probabilities proportional to the local density of agents belonging to a community choosing a given option ($\sigma_A + \sigma_B + \sigma_{AB} = 1$)[36]:

$$p_{A\to AB} = \frac{1}{2}\sigma_B, \qquad p_{B\to AB} = \frac{1}{2}\sigma_A\,; \tag{1}$$

$$p_{AB\to B} = \frac{1}{2}(1-\sigma_A), \qquad p_{AB\to A} = \frac{1}{2}(1-\sigma_B). \tag{2}$$

Equation (1) gives the probabilities for an agent i to move away from a single-option community, A or B, to the AB community. They are pro-

portional to the density of agents in the opposed single-option state in the neighbourhood of i. On the other hand, equation (2) gives the probabilities for an agent to move from the AB community towards the A or B communities. They are proportional to the local density of agents with the option to be adopted, including those in the AB state ($1-\sigma_j = \sigma_i + \sigma_{AB}$, i, j=A,B). It is important to note that a change from state A to state B or vice versa, always implies an intermediate step through the AB state. These dynamical rules reflect the special character of the third AB-state as one of coexisting options. They define a modification of the two state voter model to account for the AB state. For the voter model the transition probabilities are simply given by $p_{A\to B} = \sigma_B, \quad p_{B\to A} = \sigma_A$. These are equivalent to the adoption by the agents of the opinion of a randomly chosen neighbour. In our simulation we use random asynchronous node update and a unit of time includes N iterations so that each node has been updated on average once every time step.

An analysis of the mean field equations for this model shows the existence of three fixed points: two of them stable and equivalent, corresponding to consensus in the state A or B; and another one unstable, with non-vanishing values for the global densities of agents in the 3 states, A, B and AB. In order to describe the microscopic ordering dynamics, in which we take into account finite size effects and the topology of the network of interactions, we use as an order parameter an ensemble average interface density $\langle\rho\rangle$. This is defined as the density of links joining nodes in the network in different states [20, 22]. For random initial conditions $\langle\rho(t=0)\rangle = 2/3$. The decrease of $\langle\rho\rangle$ towards the value $\rho = 0$ corresponding to an absorbing state describes the coarsening process with growth of spatial domains in which agents are in the same state.

3. COARSENING IN A REGULAR LATTICE

We first consider the dynamics on a 2-dimensional regular lattice with four neighbours per node. We start from random initial conditions: random spatial distribution of 1/3 of the population in state A, 1/3 in state B and 1/3 in state AB. In Figure 1 we show the time evolution for a typical realization: state A takes over the system, while the opposite option B disappears. On the average consensus in either of the two equivalent states A or B is reached with probability 1/2. We observe an early very fast decay of the interface

density and of the total density of agents in the state AB, Σ_{AB}, followed by a slower decay corresponding to the coarsening dynamical stage. This stage lasts until a finite size fluctuation makes one of the sates A or B dominate, and the density of AB agents disappears together with the density of agents in the option (A or B) that vanishes.

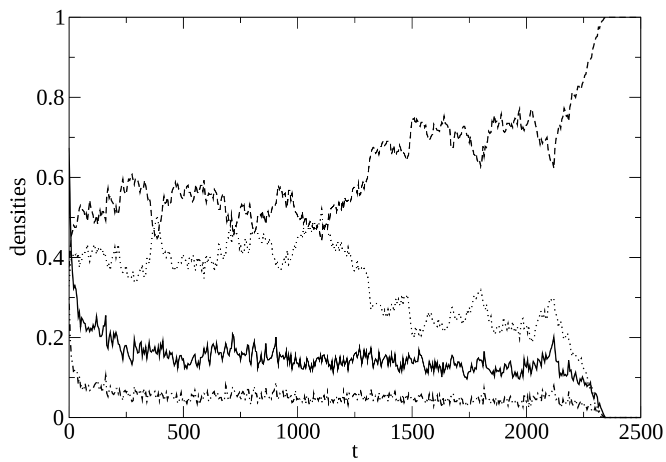

Figure 1: Time evolution of the total densities of agents in the three states, Σ_i (i= A, B, AB), and the interface density, ρ. One realization in a population of $N = 400$ agents. From top to bottom: Σ_A, Σ_B, ρ, Σ_{AB}.

In Figure 2 we show the time evolution of the interface density and of the total density of AB agents, averaged over different realizations. For the relaxation towards one of the absorbing states (dominance of either A or B) both the average interface density and the average density of AB agents decay following a power law with the same exponent, $\langle \rho \rangle \sim \langle \Sigma_{AB} \rangle \sim t^{-0.45}$. This indicates that the evolution of the average density of the AB agents is correlated with the interface dynamics. Several systems sizes are shown in order to see the effect of finite size fluctuations. During the coarsening stage describe by the power law behavior, spatial domains of the A and B community are formed and grow in size. Eventually a finite size fluctuation occurs (as the one shown in Figure 1) so that the whole system is taken to an absorbing state in which there is consensus in either the A or B option. The time scale to reach the absorbing state can be estimated to scale as $\tau \sim N^2$

since at that time $\langle\rho\rangle \sim 1/N$. During the coarsening process spatial domains of AB agents are never formed. Rather, during an early fast dynamics AB agents place themselves in the boundaries between A and B domains. This explains the finding that the density of AB agents follows the same power law than the average density of interfaces. We have also checked the intrinsic instability of an AB community: an initial AB domain disintegrates very fast into smaller A and B domains, with AB agents just placed at the interfaces. The role of third AB state is therefore identified as a mechanism to modify the dynamics of the interface.

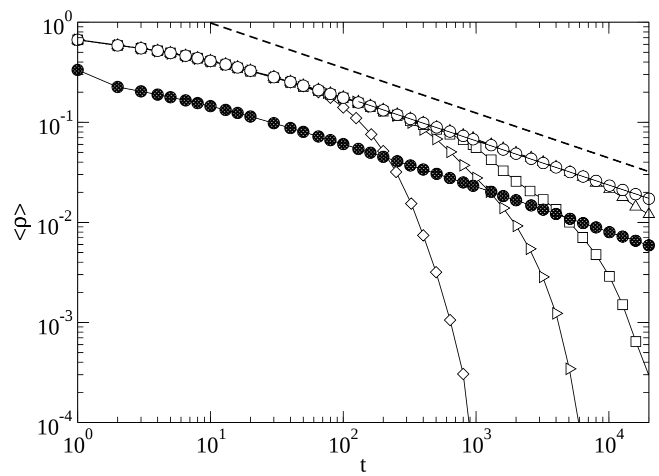

Figure 2: Time evolution of the averaged interface density $\langle\rho\rangle$ in a 2-dimensional regular lattice for different system sizes. From left to right: $N = 10^2$, 20^2, 30^2, 100^2, 300^2 agents (empty figures). The averaged global density of AB agents, $< \Sigma_{AB} >$, for $N = 300^2$ agents is also shown (filled circles). Averaged over 100-1000 realizations depending on the system size. Dashed line for reference: $\langle\rho\rangle \sim t^{-0.45}$

Our result for the growth law of the characteristic length of a A or B domain is compatible with the well known exponent 0.5 associated with domain growth driven by mean curvature and surface tension reduction observed in SFKI models. However, systematic deviations from the exponent 0.5 are observed, which can be due to non trivial logarithmic corrections. In

3-dimensional lattices, we also find an exponent close to 0.5 which substantiates the claim that curvature reduction is the dominant mechanism at work. Still, the model analyzed here is a modification of the two state voter model for which coarsening in a $d = 2$ square lattice occurs by a different mechanism, interfacial noise, such that $\langle \rho \rangle \sim (\ln t)^{-1}$ [20, 19]. For a finite system the time to reach an absorbing state scales as $\tau \sim N \ln(N)$ [37, 16]. Therefore, introduction of the AB state, in spite of the small number of agents surviving in that state, implies a nontrivial modification of the dynamics. Indeed, in our simulations we observe the formation of well defined interfaces between A and B domains, populated by AB agents, that evolve by a curvature driven mechanism. On the qualitative side, the inclusion of the AB agents gives rise to a much faster coarsening process but it also favors a longer dynamical transient in which domains of the two competing options coexist (larger lifetime time to reach the absorbing state for large fixed N).

A natural question that these results pose is if the crossover from interfacial noise dynamics of the voter model to curvature driven dynamics is generic for any structural modification of the voter model. To check this idea we have considered the coarsening process in a 2-dimensional lattice in which agents can choose between two excluding options (states A and B) and the dynamics is as defined above but with transition probabilities:

$$p_{A\to B} = \sigma_B - \epsilon \sin 2\pi\sigma_B, \quad p_{B\to A} = \sigma_A - \epsilon \sin 2\pi\sigma_A, \quad \epsilon \leq \frac{1}{2\pi} \tag{3}$$

The parameter ϵ measures the strength of the term that perturbs the interaction rules of the voter model. This perturbation of the voter model implies that the probability of changing option is no longer a linear function of the density of neighbouring agents in the option to be adopted. With the perturbation term chosen here there is a nonlinear reinforcing (of order ϵ) of the effect of the local majority: the probability to make the change $A \to B$ is larger (smaller) than σ_B when $\sigma_B > 1/2$ ($\sigma_B < 1/2$). In particular, we note that for $\epsilon \neq 0$, the conservation law of the ensemble average magnetization, a characteristic symmetry of the voter model, is no longer fulfilled. For later comparison we recall that in the zero-temperature SFKI the local majority determines, with probability one, the change of option: $p_{A\to B} = 1(0)$ if $\sigma_B > 1/2(\sigma_B < 1/2)$.

Our results for the exponent x in a power law fitting $\langle \rho \rangle \sim t^{-x}$ for the modified voter model defined by eq. (3) [38] are shown in Fig. 3 for different

values of ϵ [39]. For very small values of ϵ we observe an exponent $x \sim 0.1$ compatible with the logarithmic decay ($\langle\rho\rangle \sim (\ln t)^{-1}$) of the voter model, as obtained in [16]. However, for small, but significant values of ϵ there is a change to a value $x \sim 0.5$ associated with curvature driven coarsening.

We conclude that a small arbitrary perturbation of the transition probabilities of the voter model dynamics leads to a new interface dynamics, equivalent to the one found in Section 2 by including a third state where options are non-excluding. This indicates that voter model dynamics is very sensitive to perturbations of its dynamical rules.

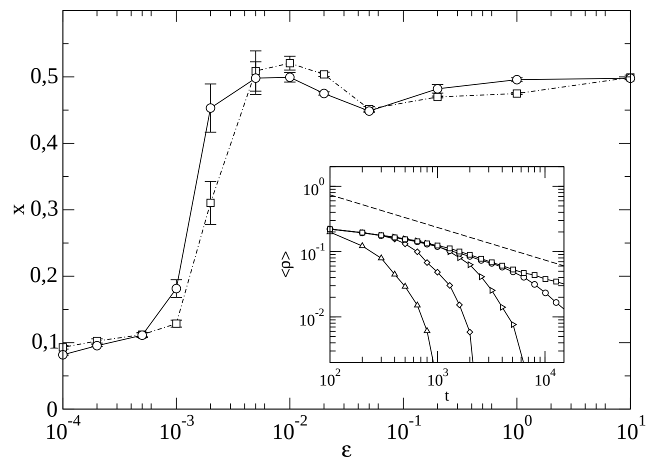

Figure 3: Characteristic coarsening exponent x ($\langle\rho\rangle \sim t^{-x}$) for the modified voter model (eq.3) [38] as a funtion of the perturbation parameter ϵ. From left to right, systems of sizes $N = 400^2$ (averaged over 50 realizations), 300^2 (averaged over 75 realizations). Inset: time evolution of the average interface densitiy, for $\epsilon = 0.01$. From left to right: $N = 20^2$, 50^2, 100^2, 200^2, 400^2 agents. Given a value of ϵ, for large enough system sizes a power law for the average interface density decay is found. Dashed line for reference: $\langle\rho\rangle \sim t^{-0.5}$

4. COARSENING IN A SMALL WORLD NETWORK

We next consider the dynamics of the model defined in Sect.2 on a small world network constructed following the algorithm of Watts & Strogatz [35]:

starting from a two dimensional regular lattice with four neighbours per node, we rewire with probability p each of the links at random, getting in this way a partially disordered network with long range interactions throughout it.

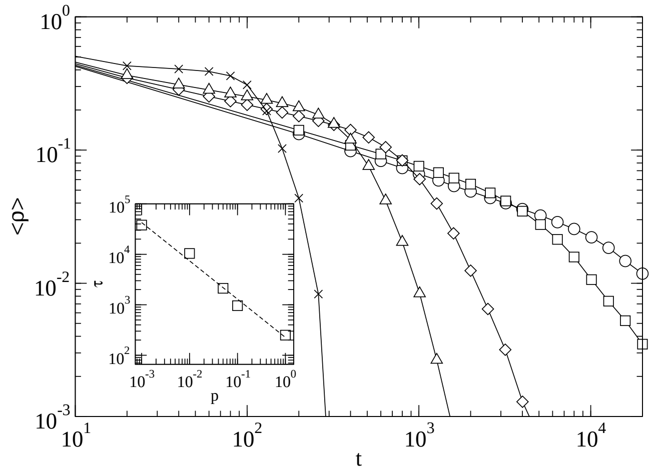

Figure 4: Time evolution of the average interface density $\langle\rho\rangle$ in small world networks with different values of the rewiring parameter p. From left to right: p=1.0, 0.1, 0.05, 0.01, 0.0. For comparison the case $p = 0$ for a regular network and the case $p = 1$ corresponding to a random network are also shown. The inset shows the dependence of the characteristic lifetime τ with the rewiring parameter p. The dashed line corresponds to the power law fit $\tau \sim p^{-0.76}$. Population of 100^2 agents, averaged over 500 realizations.

In Figure 4 we show the evolution of the average interface density for different values of p. As for the regular lattice we also observe here a dynamical stage of coarsening with a power law decrease of $\langle\rho\rangle$ followed by a fast decay to the A or B absorbing caused by a finite size fluctuation. During the dynamical stage of coarsening, the A and B communities have similar size, while the total density of AB agents is much smaller. In the range of intermediate values of p properly corresponding to a small world network, increasing the rewiring parameter p has two main effects: i) the coarsening process is notably slower; ii) the characteristic time of the dynamics τ, which we define as the time when $\langle\rho\rangle$ sinks below a given small value, drops follow-

ing a power law (inset of Figure 4): $\tau \sim p^{-0.76}$, so that the absorbing state is reached much faster as the network becomes disordered.

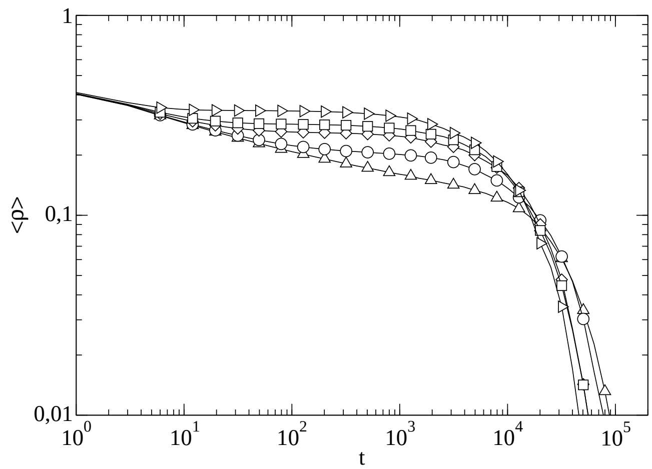

Figure 5: Time evolution of the average interface density $\langle\rho\rangle$ for the voter model in a small world network with different values of p. From up to bottom, p=1.0, 0.1, 0.05, 0.01, 0.0. Population of 100^2 agents, averaged over 900 realizations.

To understand the role of the AB state in the ordering dynamics in a small world network, the results of Fig. 4 should be compared with the ones in Fig. 5 for the two state voter model in the same small world network [40]. In contrast with the model with two non-excluding options (Section 2), moderate values of p stop the coarsening process leading to dynamical metastable states characterized by a plateau regime for the average interface density [21, 22]. However the lifetime of these states is not very sensitive to the value of p, with the characteristic time of the dynamics being just slightly smaller than the one obtained in a regular lattice ($p = 0$). This is a different effect than the strong dependence on p found for these characteristic times when AB agents are included in the dynamics. Comparing the results of Figs. 4 and 5 for a fixed intermediate value of p, we observe that including AB agents in the dynamics on a small world network of interactions allows the coarsening process to take place, and it also produces an earlier decay to

the absorbing state.

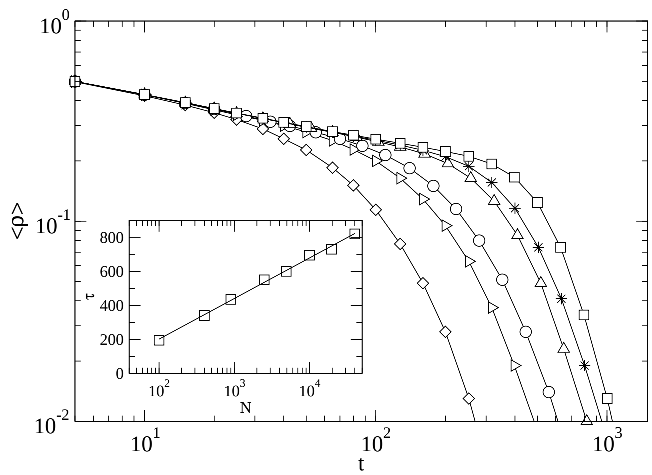

Figure 6: Time evolution of the averaged interface density, $\langle\rho\rangle$, for different values of the population size (N) in a small world network with $p = 0.1$. $N = 10^2$, 20^2, 30^2, 70^2, 100^2, 200^2 from left to right. Averaged over 1000 realizations in 10 different networks. Inset: dependence of the characteristic time τ (time when $\langle\rho\rangle$ sinks below a given small value; 0.03 in this figure) with the system size: $\tau \sim \ln(N)$.

System size dependence for a fixed value of the rewiring parameter p is analyzed in Figure 6. We observe that the initial stage of coarsening process is grossly independent of system size, but the characteristic time of the dynamics scales with the system size N as $\tau \sim \ln(N)$. For the two state voter model $\tau \sim N$ [22]. Therefore the faster decay to the absorbing state caused by the AB agents in a large system interacting through a small world network is measured by the ratio $\frac{\tau_{AB}}{\tau_{voter}}|_{SW} \sim \frac{\ln(N)}{N}$. This faster decay is the inverse than the one found for the regular lattice where the same ratio of time scales is $\frac{\tau_{AB}}{\tau_{voter}}|_{Lattice} \sim \frac{N^2}{N\ln(N)}$.

5. SUMMARY AND CONCLUSIONS

We have studied the nonequilibirum transient dynamics of approach to the absorbing state for a modified voter model defined in Sect. 2 in which

the interacting agents can be in either of two equivalent states (A or B) or in a third mixed state (AB). A global consensus state (A or B) is reached with probability one. A domain of agents in the AB state is not stable and the density of AB-agents becomes very small after an initial fast transient. In spite of these facts, the AB-agents produce an essential modification of the processes of coarsening and domain growth, changing the interfacial noise dynamics of the voter model into a curvature driven interface dynamics characteristic of two-option models based on local majorities updating rules. We have argued that this effect is generic for small structural modifications of the random imitation dynamics of the voter model. We have also considered the effect of the topology of the network of interactions studying our dynamical model in a small world network. While for the original voter model the small world topology results in long lived metastable states in which coarsening has become to a halt [21, 22], the AB-agents restore the processes of coarsening and domain growth. Additionally, they speed-up the decay to the absorbing state by a finite size fluctuation. We obtain a characteristic time that scales with system size as $\tau \sim \ln N$ to be compared with the result $\tau \sim N$ for the voter model.

From the point of view of recent studies of linguistic dynamics the modified voter model allowing for two non-exluding options, is an extension of the microscopic version [16] of the Abrams-Strogatz model [7] for two socially equivalent languages, to include the effects of bilingualism (AB-agents) [13] and social structure. Within the assumptions and limitations of our model, our results imply that bilingualism and small world social structure are not efficient mechanisms to stabilize language diversity. On the contrary they are found to ease the approach to absorbing monolingual states by an obvious effect of smoothing the communication across linguistic borders.

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[36] The prefactor 1/2 comes from the original model [13], when taking into account non-equivalent options A and B. Here it can be interpreted as an inertia that limits the maximum probability for changing sate to 1/2 setting a microscopic time scale.

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[38] It is possible to extend the perturbation of the voter model defined by eq. (3) letting ϵ to take any positive value, and choosing a modified transition probabilitiy defined as follows: $p_{A\to B}$ as given by eq. (3) for values of ϵ such that $0 \leq p_{A\to B} \leq 1$; $p_{A\to B} = 0(1)$ for values of ϵ such that eq. (3) gives $p_{A\to B} < 0(> 1)$. The limit $\epsilon \to \infty$, corresponds to the step-function transition probability of the SFKI model at T=0.

[39] In these simulations we have taken a 2-dimensional lattice with eight neighbours per node so that more possible values are allowed for the perturbation term in eq. (3).

[40] Note that the small world network considered in [21] is obtained by a rewiring process of a $d = 1$ regular lattice.